SYSTEM OF PROGRAMS FOR ANALYSIS OF THREE-DIMENSIONAL SHEAR WALL STRUCTURES

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SUMMARY
The paper presents a system of programs for calculating stresses and displacements in three-dimensional shear wall structures with uniform properties throughout the height. The analysis is carried out on the basis of the continuous connection method. The system allows for considering lateral and vertical loads, arbitrarily located in the plan and arbitrarily distributed along the height. The system is user-oriented and inexpensive in operation. Two numerical examples are given in the paper.

1. INTRODUCTION
In multi-storey buildings, lateral loads that arise as a result of winds and earthquakes are often resisted by a system of shear walls acting as vertical cantilevers. Such walls are usually perforated by vertical bands of openings which are required for doors and windows to form a system of coupled shear walls (Figure 1).

The analysis of shear wall structures may be performed by using either discrete or continuous methods. In the continuous approach, the horizontal connecting beams, floor slabs, and vertical joints are substituted by continuous connections. The fact that floor slabs have infinite in-plane stiffness is the main assumption used in this method. Shear walls are considered as thin-walled cantilever beams. This method is appropriate when the properties of walls and connections are sensibly uniform throughout the height.

Equation formulations for a three-dimensional continuous model have been presented by Biswas and Tso,1 Coull and Irwin,2 Danay et al.,3 Jendele and Sejnoha,4 Laredo,5 Lewicki et al.,6 Petersson,7 Rapp and Wrześniowski9 and Rosman.1 In the papers by Fransson,10 Lis,11 and Petersson,7 models were used that are continuous along the height and discrete in the transverse direction. The use of the continuous model allows the avoidance of some difficulties concerning the great number of unknowns and the ill-conditioning of the problem for slender structures, which occurs during the analysis according to the discrete model (Bauer et al.12). The program system presented in the present paper has been performed on the basis of the continuous model, using as unknowns the intensity functions of shear forces in continuous connections which substitute connecting beam bands and vertical joints.

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2. GOVERNING EQUATIONS

In the force formulation used while creating the system of programs, the governing differential equations have the following form:

\[
B n''(z) - A n(z) = f(z) \quad (1)
\]
\[
v''(z) = f_p(z) - V_n n(z) \quad (2)
\]

The boundary conditions for (1) and (2) can be stated as follows:

\[
n(0) = w, \quad n'(H) = 0 \quad (3)
\]
\[
v(0) = 0, \quad v'(0) = 0, \quad v''(H) = 0 \quad (4)
\]

The matrices appearing in the above relations are described by the following formulae:

\[
A = S^T_E K_E S_E - C_N L (L^T K_L L)^{-1} L^T C_N \quad (5)
\]
\[
f(z) = (S^T_E K_E S_E - C_N L (L^T K_L L)^{-1} L^T C_R) n(z) + C_N L (L^T K_L L)^{-1} t_k(z) \quad (6)
\]
\[
f_p(z) = (L^T K_L L)^{-1} t_k(z) - (L^T K_L L)^{-1} L^T C_R n(z) \quad (7)
\]
\[
V_N = (L^T K_L L)^{-1} L^T C_N \quad (8)
\]
\[
w = -B^{-1} S^T_E z_0 \quad (9)
\]
where the following notation applies.

\[ z \in [0, H] \]

- **H** construction height
- \( n_w \) number of continuous connections
- \( n_e \) number of shear walls
- \( n_v \) number of vertical loads

**n(z)** vector containing unknown functions of the shear force intensity in continuous connections which substitute connecting beam bands and vertical joints

**B** \( n_w \times n_w \) diagonal matrix containing continuous connection flexibilities

**v(z)** vector containing the functions of horizontal displacements of the structure, measured in the global coordinate system \( OXYZ \), which may be chosen arbitrarily, \( v(z) = \text{col}(v_x(z), v_y(z), \phi(z)) \)

**S_E** \( n_e \times n_e \) boolean matrix, related to the interaction between shear walls and continuous connections

**K_S** \( n_e \times n_e \) diagonal matrix, \( K_S = \text{diag}(1/E_{A_1}) \)

**K_Z** \( 3n_e \times 3n_e \) matrix containing transverse stiffness of shear walls in local coordinate systems, i.e. systems of principal axes of the shear walls, \( K_Z = \text{diag}(-EI_{y_1}, ..., -EI_{y_1}, ...) \)

**C_N** \( 3n_e \times n_e \) matrix containing the coordinates of the points of contraflexure in connections in the local coordinate systems \( C_N = (C_{N_1x}, C_{N_1y}, C_{N_1z})^T \)

**L** \( 3n_e \times 3 \) matrix of coordinates transformation from the global coordinate system \( OXYZ \) to the local system of axes

**S_R** \( n_e \times n_e \) boolean matrix, related to the action of exterior vertical loads on shear walls

**C_R** \( 3n_e \times n_e \) matrix containing the coordinates of points of application of vertical loads in local coordinate systems

**n_R(z)** the vector of the functions of vertical loads

**t_k(z)** the vector of the functions of shear forces and a torque due to the action of lateral loads, \( t_k(z) = \text{col}(t_x(z), t_y(z), m_k(z)) \)

**z_0** the vector containing given settlements of shear walls

All necessary matrices and vectors are automatically computed in the system programs.

### 3. METHOD OF SOLUTION

Searching for solutions of a homogeneous equation corresponding to equation (1) in the form

\[ n(z) = x \exp(\sqrt{\lambda} z) \]

results in a generalized eigenvalue problem:

\[ (A - \lambda B)x = 0 \tag{10} \]

The matrix **B** is diagonal and positive definite, while matrix **A** is symmetrical and positive semidefinite. These features have been used to transform the generalized eigenvalue problem (10) to a standard form

\[ (P - \lambda I)y = 0, \quad P = B^{-1/2}AB^{-1/2} \tag{11} \]

where **P** is a symmetrical matrix.

The eigenvalues and eigenvectors of matrix **P** are computed by a set of procedures realizing the Householder's tridiagonalization and the QL algorithm, taken from [17]. The determined eigensystem fulfills the orthogonality relations

\[ Y^T Y = I, \quad Y^T P Y = \text{diag}(\lambda_i), \quad (i = 1, \ldots, n_w) \tag{12} \]
so that the system of equations (1) can be uncoupled into \( n \) second-order differential equations of the form

\[
g'_i(z) - \lambda_i g(z) = F_{hi}(z)
\]

with the boundary conditions

\[
g_i(0) = Y_i^T B^{1/2} w, \quad g_i'(H) = 0
\]

where

\[
g_i(z) = Y_i^T B^{1/2} n(z), \quad F_{hi}(z) = Y_i^T B^{-1/2} f(z)
\]

In the system, a polynomial form of the function \( f(z) \) has been used:

\[
f(z) = F w_g(z), \quad w_g(z) = \text{col}(z^0, \ldots, z^{s-1})
\]

A procedure for least squares fit by orthogonal polynomials for the approximation of the function of loads has been applied.

The solutions of the equations (13), corresponding to non-zero eigenvalues, \( \lambda_i \neq 0 \), are computed from

\[
g_i(z) = C_{1i} \exp(\sqrt{\lambda_i} z) + C_{2i} \exp(-\sqrt{\lambda_i} z) + r_i w_g(z)
\]

where

\( r_i[1, \ldots, s] \) particular solution coefficients, calculated by the indeterminate coefficient method,

\( C_{1i}, C_{2i} \) integration constants, computed from boundary conditions (14).

The solutions corresponding to zero eigenvalues can be obtained through integration of (13) with the boundary conditions (14).

Owing to a consistent uncoupling of the system of the differential equations (1), the required size of computer memory decreases.

Functions of horizontal displacements \( u(z) \) and their derivatives are computed from formulae obtained by integration of (2) with the boundary conditions (4).

4. SYSTEM OF PROGRAMS

The static analysis of multistorey buildings (SAMB) system has been realized for the Odra 1300 and ICL 1900 series computers. With an available 64 K 24-bit words memory and discs or magnetic tapes, it is possible to analyse constructions in which the number of connections does not exceed 100. The number of shear walls, storeys, and load patterns is practically unlimited. Appropriate macro-commands have been prepared for the George 3 operating system to enable a simple use of the system. At present, the SAMB system is being transformed for microcomputers.

Computations are performed in the following steps:

program DAT1: input, data checking and error diagnostics for data describing the shear wall structure,

program DAT2: input, data checking and error diagnostics for static load and settlement patterns,

program ECHO: data printout,

program CHAR: computation of geometric properties of shear walls,

program COEF: computation of matrices \( A \) and \( B \) dependent on the structure—equation (1),
program LOAD: computation of static load vectors \( f(z) \) — equation (1).

program EIG1: finding the solution of the generalized eigenproblem \( (A - \lambda B) = 0 \), corresponding to the homogeneous differential equation.

program PSOL: calculation of coefficients for particular solutions and of integration constants.

program SSOL: calculation of the values of \( n(z) \) and \( v''(z) \) for successive load patterns.

program DISP: calculation of values of integrals \( \int_0^n n, \int_0^n v \) and functions \( v'', v', v \).

program SSTR: calculation of normal forces and stresses \( \sigma_z, \tau, \sigma_\parallel, \sigma_\perp \) in shear walls.

program SFOR: calculation of internal forces \( M_X, M_Y, B, T_X, T_Y, M_v, M_{sv} \) in shear walls.

program GROUP: modification of tables \( v, \sigma \) and of internal forces tables: from tables for load schemes to tables for shear walls.

program EXTR: calculation of extreme values of results for given variants of extremes.

program RES2: printout of results.

5. DATA AND RESULTS

The description of data for computations is given in a form similar to the problem-oriented language. The data are grouped in the following way:

(a) identifiers (they include, among other things, the name of the user, the number of the data set, etc.);

(b) general data (e.g. modules of elasticity, construction height, etc.);

(c) coordinates of characteristic points in the chosen global coordinate system \( 0XYZ \);

(d) specification of shear walls, vertical bands of connecting beams, and flexible joints;

(e) specification of each story, including the heights of its bottom and top from the base;

(f) specification of static load and settlement patterns;

(g) ordinates of height;

(h) control variables (i.e. the numbers controlling the range of printed results and the form of printouts).

The shear walls can have any cross-sectional shapes, with arbitrary locations and orientations in the plan. Both lateral and vertical loads may be arbitrarily distributed along the height. Lateral loads are described by the intensities of loads \( q_X(z), q_Y(z), m_D(z) \) for characteristic ordinates, and by values of concentrated top loads. Vertical loads can act along vertical lines at arbitrary locations in the plan. They are designed by the values of loads accumulated from the heights of successive storeys.

The formal correctness of input data is checked by the first two programs.

All the data for the computations are given in an arbitrarily chosen global axis system \( 0XYZ \) (\( z = 0 \) at the level of the structure fixing at the base). Computations are performed and results are presented with reference to this system.

The final results are presented in tables. They include:

(i) geometrical characteristics of shear walls cross-sections;

(ii) displacements;

(iii) shear forces and bending moments in connecting beams;
(iv) normal, shear, and principal stresses at some characteristic points of a structure's horizontal cross-section;
(v) internal forces in shear walls optional.

The results are computed for arbitrary ordinates of height given in the input data. All groups of results can be obtained for every load pattern and for given variants of extremes.

6. NUMERICAL EXAMPLES

The computation results compared with the results presented in the papers by Biswas and Tso,\textsuperscript{1} Cholewicki and Kaproni,\textsuperscript{13} Danay \textit{et al.},\textsuperscript{3} Fransson,\textsuperscript{10} Gałkowski and Samborski,\textsuperscript{14} Jendele and Sejnoha,\textsuperscript{4} Lis,\textsuperscript{11} Petersson,\textsuperscript{7} and others proved the correctness of the system. Also, there is a good agreement of our results and those obtained by model testing and by finite element analysis.\textsuperscript{7,15}

\textit{Example 1}

To illustrate the correctness of the work of the programs, a 20 storey shear wall building considered by Danay \textit{et al.}\textsuperscript{3} is chosen. The plan of the building is shown in Figure 2; it consists of ten shear walls and two vertical bands of connecting beams.

The storey height is 3.2 m. The shear wall thickness is 0.2 m. The effective width of the connecting beams is taken to be 0.2 m and their height is taken to be 0.6 m. Values of the modulus of elasticity, \( E \), and shear modulus, \( G \), are taken at 1.0 and 0.5, respectively in the calculations. The columns have no significant flexural carrying capacity.

An equally distributed 4.0 T m\(^{-1}\) lateral load is applied along the \( Y \)-axis. The units used by Danay \textit{et al.}\textsuperscript{3} are not changed. In the work of Danay \textit{et al.}\textsuperscript{3} the unknowns of the problem are three functions of horizontal rigid plane displacement of the structure and an axial displacement function for each shear wall connected by beams. Danay obtained a numerical solution of the differential equations system, using the infinite polynomial series.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{example_building_plan.png}
\caption{Plan of example building}
\end{figure}
Figure 3. Displacements and shear force intensity functions for connecting beam bands.
For the structure described above, the system of differential equations (1) presented in this paper includes two equations. The time of computation for its solution was 7 s (CPU) on an Odra 1305 computer. Figure 3 shows the diagrams of displacements, rotation, and of functions of shear force intensity in connecting beam bands. The values of Danay et al. are given in brackets. There is good agreement between our results and those obtained by Danay et al.

Example 2

Another example shows how the SAMB system is used in a design process. The example is the monolithic core of a building that was constructed in Poznań. The plan of the structure, with the numbering of walls, connecting beams, and characteristic points located on the wall axes is shown in Figure 4. The construction height is $H = 71.4$ m, the storey height is 3.3 m. The properties of the material are taken to be $E = 26$ GPa and $G = 11.05$ GPa.

The flexibility of the connecting beams is computed assuming the fixing of their ends. In order to include the effect of beam–shear wall junction deformations, the $K$ coefficients have been
Table I

<table>
<thead>
<tr>
<th>Number of beam</th>
<th>1, 2</th>
<th>3, 8</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_n$</td>
<td>1.85</td>
<td>1.25</td>
<td>1.25</td>
<td>1.25</td>
<td>1.05</td>
<td>0.98</td>
<td>0.98</td>
<td>1.25</td>
</tr>
<tr>
<td>$K$</td>
<td>1.53</td>
<td>1.04</td>
<td>1.55</td>
<td>1.59</td>
<td>1.13</td>
<td>0.84</td>
<td>1.23</td>
<td>0.84</td>
</tr>
</tbody>
</table>

introduced. $K$ coefficients increase the effective length of beams, according to the formula

$$l_0 = l_n + Kh_n$$

where $l_n$ is the clear span of a beam and $h_n$ is the depth of a beam.

Table I gives the depths of beams and the $K$ coefficients used in the calculations, determined from the results of finite element analysis (Reference 16).

Next, we present results obtained for a selected load pattern, for which the lateral load distribution $q_Y(z)$ is presented in Figure 5(a). The torque is $m_b(z) = q_Y(z) \times 6.2$ m. Figure 5 shows the functions of shear force intensity in two chosen bands of connecting beams, and the horizontal displacements of the point number 1, measured in the global coordinate system $0XYZ$.

Figure 5. Load and results for example structure (a) lateral load $q_Y(z)$; (b) shear force intensity functions in two bands of connecting beams; (c) horizontal displacements of the structure
The vertical normal stresses at the base for the same load pattern are shown in Figure 6. The Odra 1305 computer processing time for this load pattern was 33 s (CPU).

7. CONCLUSIONS

The paper presents the system of programs for three-dimensional linear static analysis of multistorey shear wall buildings. The system was thoroughly tested to prove the correctness of its work. The use of the system allows one to perform effective, low cost analysis of stresses and displacements in non-symmetrical structures with shear walls and connecting beams arbitrarily located in the plan.

REFERENCES