Integrated System for Analysis of Shear Wall Tall Buildings

Jacek Wdowicki
Elżbieta Wdowicka
Tomasz Blaszczyński

Lateral loads that arise from effects of wind and earthquakes, are often resisted by a system of shear walls acting as vertical cantilevers. Such walls are usually perforated by vertical bands of openings, which are required for doors and windows to form a system of coupled shear walls, Fig. 1.

There are basically two approaches for analysis of coupled shear walls: discrete and continuous. In the continuous approach, which has been widely used for shear walls uniform along the height, the discrete set of connecting beams is replaced by a continuous medium of equivalent properties. In that approach it has been shown that, the uniform braced frame, rigid frame and coupled wall structures belong to a group of cantilevers based on their bending and shearing characteristics (Stafford Smith et al., 1981; Coull and Wahab, 1993).

From Coull (1988) it is known that no general computer packages have ever been produced for the use of continuum methods, and individuals have produced their own programs for the solution of specific structural forms.

The design of multistorey buildings for earthquake ground motion involves dynamic analysis of structure. In the case of moderate ground motion the response of structure may be assumed linear. A continuous method for determination of linear response to an impact load and earthquake accelerations was presented by Danay and others (1975). Tso and others (1977) proposed procedure based on the response spectrum technique to estimate the seismic response of coupled shear walls. A response spectrum approach was also used in the program for discrete
analysis of frame-shear wall structure by Humar and Khandoker (1980). A computer program for the static and free-vibration analyses of frame-shear wall structure by continuum approach was listed by Swaddiwudhipong and others (1986). The program for frequency analysis of asymmetric shear wall buildings was presented by Wdowicki (1984).

When multistory buildings with non-coincident centres of mass and stiffness are subjected to earthquake ground motion they respond in coupled lateral and torsional vibrations. Torsional coupling may induce significant amplification of inertial force resulting from earthquakes (Chandler and Hutchinson, 1987), and it is felt to be strongly influenced by building asymmetry and the twisting or torsional motion of the ground, which is shown by Hart and others (1975).

Fig. 1 3-D shear wall structure: 1- 3-D shear wall, 2- continuous connection, 3- floor
In the present paper an integrated system of programs for linear static and dynamic analysis of 3-D shear wall structures by means of the response spectrum method is presented.

**MODEL AND THEORETICAL BACKGROUND OF ANALYSIS**

A single wall or a group of walls joined in a monolithic way creates a 3-D shear wall. The structure may consist of an unlimited number of two and three-dimensional shear walls freely distributed in a plan. The same height shear walls are considered. They may be joined by lintel's bands or by flexible joints. The structural properties of shear walls, lintels and vertical joints are uniform along the building height. A diaphragm action of all floors is taken into consideration as the effect of its in-plane infinite rigidity and negligible transverse one. The height to width ratio of the shear walls is such that each wall may be treated as an open thin-walled beam.

Some difficulties concerning a great number of unknowns and ill-conditioning of a problem for slender structures, which appear in the discrete model, may be avoided in a simple way using the continuous model.

**STATIC ANALYSIS**

The first subsystem is for static analysis of multistorey buildings (SAMB). The static analysis has been performed using as unknowns intensity functions of shear forces in continuous connections. In this formulation governing differential equations are as follows:

\[ Bn''(z) - An(z) = f(z) \]  

\[ v'''(z) = f'_p(z) - V_n(z) \]  

The boundary conditions for (1) and (2) can be stated as follows:

\[ n(0) = w, \quad n'(H) = 0 \]  

\[ v(0) = 0, \quad v'(0) = 0, \quad v''(H) = 0 \]

The matrices appearing in the previous relations are described by the following formulae:

\[ A = S^T_E K E S_E - C_N L (L^T K_L L)^{-1} L^T C_N \]
\[
\begin{align*}
\mathbf{f}(z) &= \left( S_0^T K_S S_R - C_N^T L (L^T K_Z L)^{-1} L^T C_R \right) \mathbf{n}_R(z) + C_N^T L (L^T K_Z L)^{-1} \mathbf{t}_K(z) \\
\mathbf{f}_p(z) &= (L^T K_Z L)^{-1} \mathbf{t}_K(z) - (L^T K_Z L)^{-1} L^T C_R \mathbf{n}_R(z) \\
V_N &= (L^T K_Z L)^{-1} L^T C_N \\
\mathbf{w} &= -B^{-1} S_E^T \mathbf{z}_0
\end{align*}
\]

where the following notation applies:

- \( z \in [0,H] \)
- \( H \) construction height
- \( n_w \) number of continuous connections
- \( n_e \) number of shear walls
- \( n_v \) number of vertical loads
- \( \mathbf{n}(z) \) vector containing unknown functions of the shear force intensity in continuous connections which substitute connecting beam bands and vertical joints
- \( \mathbf{B} \) \( n_w \times n_w \) diagonal matrix containing continuous connection flexibilities
- \( \mathbf{v}(z) \) vector containing the functions of horizontal displacements of the structure, measured in the global coordinate system \( OXYZ \), which may be chosen arbitrarily, \( \mathbf{v}(z) = \text{col}(v_X(z), v_Y(z), (z)) \)
- \( S_E \) \( n_e \times n_w \) Boolean matrix, related to the interaction between shear walls and continuous connections
- \( K_S \) \( n_e \times n_e \) diagonal matrix, \( K_S = \text{diag}(1/EA_i) \)
- \( K_Z \) \( 3n_e \times 3n_e \) matrix containing transverse stiffness of shear walls in local coordinate systems, i.e. systems of principal axes of the shear walls, \( K_Z = \text{diag}(-EI_{y1} ..., -EI_{x1} ..., -EI ...) \)
- \( C_N \) \( 3n_e \times n_w \) matrix containing the coordinates of the points of contraflexure in connections in the local coordinate systems
- \( C_N = (C_{N\alpha}, C_{N\nu}, C_{N\alpha}) \)\( ^T \)
- \( L \) \( 3n_e \times 3 \) matrix of coordinates transformation from the global coordinate system \( OXYZ \) to the local system of axes
- \( S_R \) \( n_e \times n_v \) Boolean matrix, related to the action of exterior vertical loads on shear walls
- \( C_R \) \( 3n_e \times n_v \) matrix containing the coordinates of points of application of vertical loads in local coordinate systems
- \( \mathbf{n}_R \) the vector of the functions of vertical loads
- \( (z) \) the vector of the functions of shear forces and a torque due to the action of lateral loads, \( \mathbf{t}_K(z) = \text{col} (t_X(z), t_Y(z), m_S(z)) \)
- \( \mathbf{z}_0 \) the vector containing given settlements of shear walls
All necessary matrices and vectors are automatically computed.

The differential equations (1) and (2) with the boundary conditions (3) and (4) are decoupled by orthogonal eigenvectors. Next, the uncoupled equations are exactly solved for the following loadcases.

**DYANMATIC ANALYSIS**

The second subsystem is an extended version of DAMB (Dynamic Analysis of Multistorey Buildings) presented by Wdowicki and Wdowicka (1991). It is for analysis of coupled torsional-flexural vibration of shear wall multistorey structures due to seismic action moreover analyses the structure as a 3-D model.

A dynamic model with masses in the form of rigid floor slabs has been adopted since over a half of building total mass is concentrated in the floor levels. Discrete masses may be free located throughout the height. Seismic response of structure is estimated by using the response spectrum technique.

The vibration of a multidegree of freedom system is described by relation:

\[
M \ddot{x} + C x + K x = F
\]  

(10)

where:

\[M\] - mass matrix,
\[C\] - damping matrix,
\[K\] - stiffness matrix,
\[x\] - d-element vector of generalized coordinates (d - number of dynamic degrees of freedom of the calculated structure).
\[F\] - d-element vector of generalized excitation forces, corresponding to generalized coordinates.

For a shear wall multistorey structure is more natural to determine the flexibility matrix \(D\) then the stiffness matrix \(K\). The vibration of a structure is described by relation:

\[
DM \ddot{x} + DC x + x = DF
\]  

(11)

The flexibility matrix is generated from the exact solution of the governing differential equation for a 3-D continuous model. Also mass matrix is generated exactly according to real distribution of walls,
connecting beams and floor slabs and including flexural and torsional inertia.

The steps involved are: 1- determination of natural frequencies and mode shapes, 2- evaluation of modal participation factors and calculation of modal loading on the structure (using an appropriate design spectrum), 3- determining a root mean square response estimate containing the contribution from the given number of nodes for various parameters of interest.

INTEGRATED SYSTEM

The system considers lateral and vertical loads, free located in plan and freely distributed along the height. It is user-oriented and not expensive in operation.

Dynamic computations determine the displacement of the vibrating structure in relation to the static equilibrium. Therefore, the total displacements of a building are algebraic sum of displacements generated by static loads and displacements computed by means of a dynamic analysis.

It is difficult to predict which displacement values, produced by pressure wind or an earthquake, will be larger in the zones characterized by a moderate degree of seismicity (rather small ones). Consequently, which one should be added to the displacements caused by dead and live loads to obtain extreme values of the displacements. The fact that the direction of a seismic wave is unknown creates additional difficulty. Thus, it is necessary to compute a dynamic response of the structure at different seismic wave directions, particularly, in the case of buildings without characteristic stiffness axes. Similar difficulties can come across when computing forces in connecting beams and stresses in the walls.

Without a computer program calculating extreme values of: displacement, shear forces in connecting beams and numerous stresses in the walls on the basis of the previously computed values for the load cases, calculations can be time-consuming and it is very easy to make a mistake in this laborious task. The difficulties described above do not occur when the integrated system is used.

The integrated system presented in the paper is created by connecting two subsystems: one for static analysis (SAMB, Wdowicki and Wdowicka (1993)) and the other one for a dynamic analysis (DAMB, Wdowicki and Wdowicka (1991)). The modules of interface, necessary for integration, do not exceed 5% of total size of the system. Apart from that only minimal changes in the existing programs are required.

The shear walls can have any cross-sectional shape and their location and orientation can be freely distributed in the plan. Lateral loads are
described by uniform loads $q_x(z)$, $q_y(z)$, $m_s(z)$ for characteristic ordinates, and values of concentrated points loads at any storey. The building can be affected by horizontal seismic waves of any direction.

The input data in a form similar to the problem-oriented language are given. They are grouped in the following way:

- job information,
- basic data (e.g. modules of elasticity, properties of material etc.),
- coordinates of characteristic points in the chosen global coordinates OXYZ,
- properties of shear walls, vertical bands and flexible joints,
- properties of floor slabs,
- specification of each storey (bottom and top distance from the base and floor slab type number,
- declaration of the design response spectrum,
- declaration of the directions of seismic waves,
- specification of the static load and settlement cases,
- control variables.

The correctness of input data is checked in the system.

The system output is presented in tables as well as in graphs and includes the following results:

- geometrical characteristics of shear wall cross-sections,
- mass characteristics of all storeys,
- eigenfrequencies and mode shapes,
- displacements,
- shear forces and bending moments in connecting beams,
- normal, shear and principal stresses in the characteristic points of a floor plan at any level,
- optional internal forces in shear walls.

The results are computed for any ordinates. All results can be obtained for each loadcase and any required combination or envelope.

The Integrated System for static and dynamic analysis of multistorey buildings is composed of twenty four modules of the first level in the form of programs compiled independently, Fig. 2.

The individual programs of the Integrated System communicate among themselves by means of one disk file, which is placed for successive calculated data dynamically by the macro command.
Fig. 2 Modules of integrated system
NUMERICAL EXAMPLES

Example 1

As an example to illustrate the correctness of the dynamic analysis a 24 storey shear wall building, considered by Tso and Biswas (1972), is used. The shear wall structure consists of five parallel pairs of coupled shear walls. The plan of coupled shear wall unit, taken into account in calculations, is shown in Fig. 3.

Fig. 3  Floor plan and coupled shear walls unit
The shear wall thickness is taken to be 9 in. and floor slabs 6 in. The storey height is 9 ft. The effective width of the floor is taken to be 4.4 ft and the weight per unit height per set of coupled shear walls is taken to be 12.6 kips/ft. The value of modulus of elasticity used in the calculations is 288000 ksf. The 1940 El Centro earthquake velocity spectra with a 5% damping are used.

The first three natural frequencies of structure computed by Integrated System are compared in Table 1 with those given by Tso and Biswas (1972), obtained by means of Rayleigh's principle. The differences are less than 1.5%.

The variation of root sum square (RSS) values of distributed shear force throughout the height differs less then 5%.

Example 2

Another example shows how the Integrated System is used in a design process. The job is the monolithic core of the tallest building in Poznań for the Academy of Economics, Fig. 4.

The plan of the core, with the numbering of walls, lintels and characteristic points located on walls is shown in Fig. 5.

The construction height is 71.4 m, a storey height is 3.3 m and the material properties are taken to be $E = 26$ GPa and $G = 11.05$ GPa.

The results obtained for a selected loadcase with lateral load distribution $q_z(z)$, are presented in Fig. 6. It shows the distribution of shear force in two chosen bands and the horizontal displacements of the point 1, measured in the global coordinate system OXYZ.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency Integrated System</th>
<th>Approx. frequency Tso and Biswas (1972)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.7037</td>
<td>0.7009</td>
</tr>
<tr>
<td>2</td>
<td>3.2246</td>
<td>3.2616</td>
</tr>
<tr>
<td>3</td>
<td>8.2326</td>
<td>8.3413</td>
</tr>
</tbody>
</table>
Fig. 4  Academy of Economics Library in Poznań
Fig. 5  Plan of the Academy of Economics Library core

Fig. 6  Loads and results for examined structure  (a) lateral load;  (b) shear force distribution;  (c) horizontal displacements of point  No 1
Example 3

As another practical application an asymmetric, 14 storey building with 13 rows of connecting beams is considered. The layout of analysed building is shown in Fig. 7.

The total height of structure is 49.1 m. The design response spectrum from 1987 Bulgarian seismic code is used. Calculations are performed for many load combinations and different structural schemes. Figure 8 presents vertical normal stresses at the base for assumed seismic wave direction.

CONCLUSIONS

The Integrated System gives a possibility to perform a full analysis of complex 3-D shear wall structures. It was thoroughly tested to prove its correctness. An application of the system allows to perform effective, low cost analysis of stresses and displacements in non-symmetrical structures with shear walls and connecting beams freely distributed in plan.
Fig. 8 Normal stresses at the base (z = 0)
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